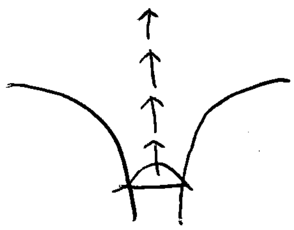
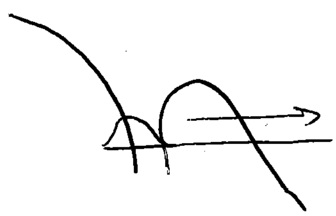
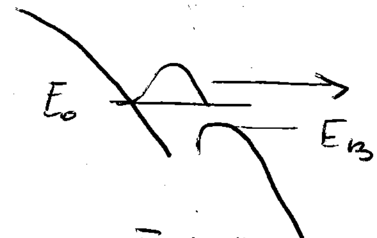
High Harmonic Generation

Ionization mechanisms (reminder from the previous lecture):

multiphoton ionization	tunneling ionization	over-barrier ionization
		
$\gamma \gg 1$	$\gamma \ll 1$	$E_0 > E_B$

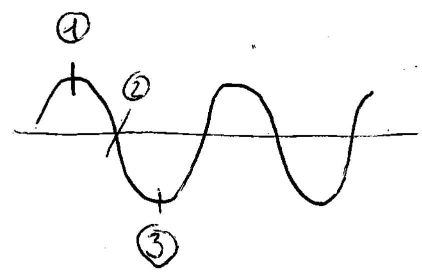
$\gamma = \dfrac{\omega}{\omega_t}$ — ω - optical period, ω_t - tunneling time

$\omega_t = \sqrt{\dfrac{I_p}{2U_p}}$ $\qquad U_p = \dfrac{E_0}{4\omega^2}$

Keldysh parameter $\qquad$ ponder motive potential

Let us consider in more details the dynamics of tunneling ionization

Laser pulse:
$\vec{E}(t) = E_0 \cdot \cos(\omega_0 t)$



• The time instance ①: the interaction potential $V(t) = V_c(x) + \vec{x}\cdot\vec{E}(t) =$

The probability of tunneling is max, because the width of the barrier is smallest,

$V(t_1) = V_c(x) + x E_0$

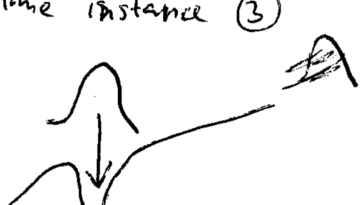
WP starts to travel from the GS to continuum

– The time instance ②

$V(t_2) = V_c(x) + 0$ (no influence of the EF)

WP from step 1 ~~turning point~~ has max. velocity

– The time instance ③ $\quad V(t_3) = V_c(x) - x\cdot E_0$



WP arrives back to the initial position and can recombine

relize of energy by rescattering, emission of photons.

Classical picture of HHG

We consider an $\bar{e}$ moving in the laser field $\vec{E}(t) = \vec{E}_0 \cos(\omega_0 t)$

Newton's second Law:

$$\frac{d\vec{v}}{dt} = -\frac{e}{m}\vec{E}(t) = -\frac{e}{m}\vec{E}_0 \cos(\omega_0 t)$$

$$\vec{v}(t) = -\frac{eE_0}{m\omega}\sin(\omega_0 t) + V_{drift}$$

$$\vec{x}(t) = \vec{x}_0 + \vec{V}_{drift}\cdot t + \frac{eE_0}{\omega^2}\cos(\omega t)$$

Let us consider a situation where the ~~free~~ electron is created in the same position $X_i = 0$ and having zero velocity $V_i = 0$. but at diffent instanee of time t_i

We can write:

$$v(t_i) \equiv 0 = -\frac{eE_0}{m\omega}\sin(\omega_0 t_i) + V_{drift}$$

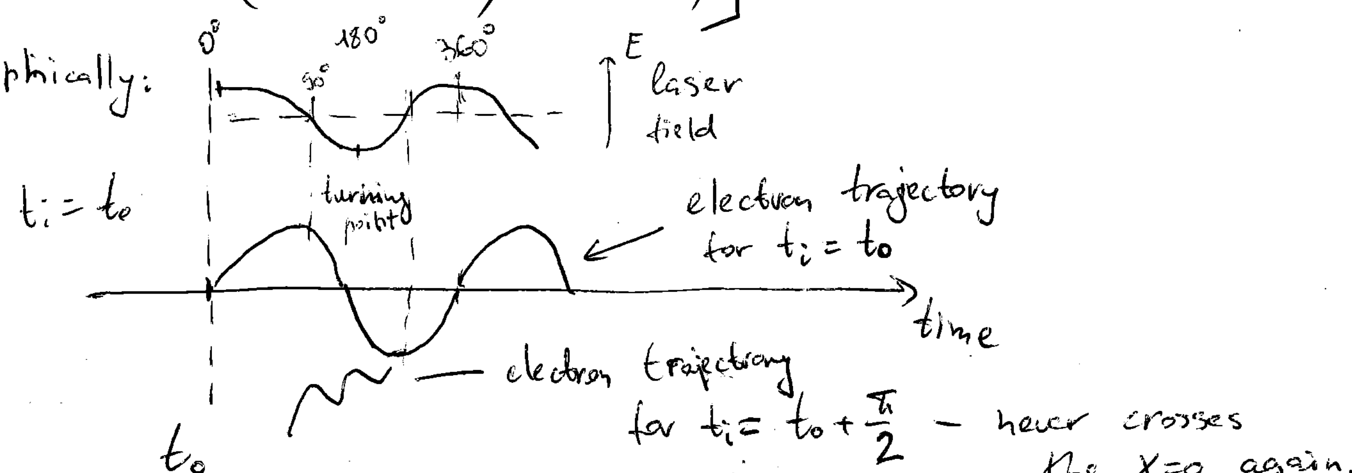
$$V_{drift} = \frac{eE_0}{m\omega}\sin(\omega_0 t_i),$$ so we obtain

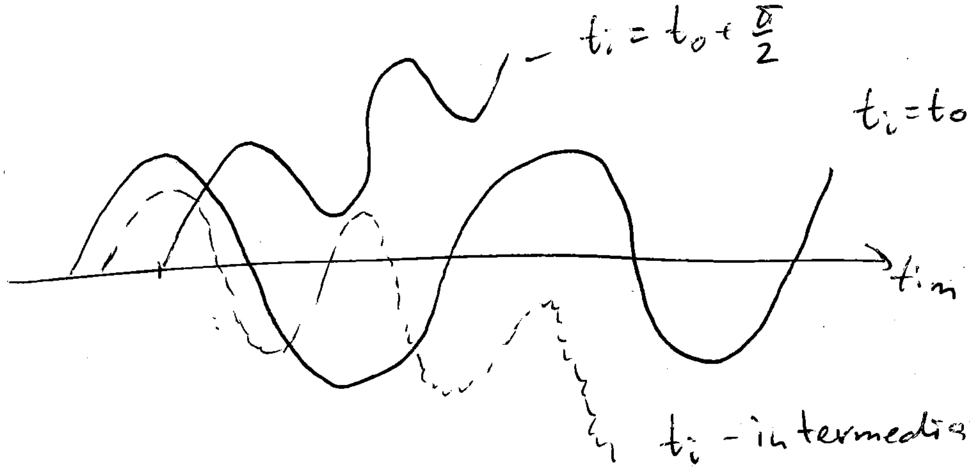
$$V(t) = -\frac{eE_0}{m\omega}\left(\sin(\omega_0 t) - \sin(\omega_0 t_i)\right)$$ – velocity of the electron at time t if it was created at time t_i at position $X=0$ and with zero velocity $v=0$.

The trajectory of such electron is:

$$X(t) = -\frac{eE_0}{m\omega}\Big[\left(\cos(\omega_0 t) - \cos(\omega_0 t_i)\right) + (\omega_0 t - \omega_0 t_i)\sin(\omega_0 t_i)\Big]$$

Graphically:





what is the ~~time~~ time when electron comes back again?

$$x(t_r) = 0,$$

t_r - recombination time.

$x(t_r)=0$ doesn't have analytical solution but it can be solved numerically. The recombination can happen only if $0 \leq \omega_0 t_i^* \leq 80^\circ$, and $180^\circ \leq \omega_0 t_i \leq 260^\circ$

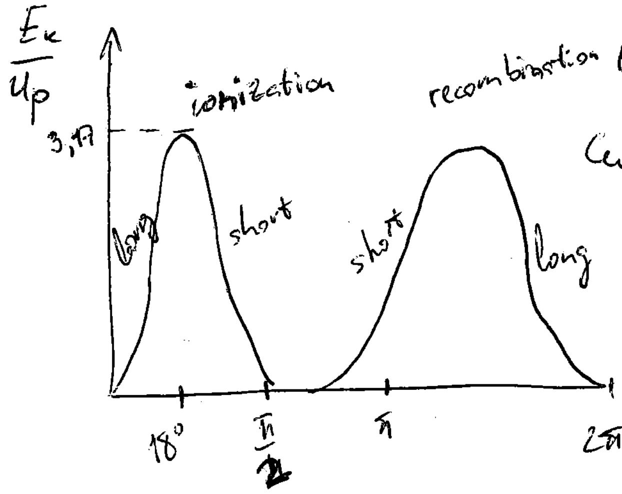
The solution can be fitted by the expression:

$$\frac{t_r}{T_0} = \frac{1}{4} - \frac{3}{2\pi} \sin^{-1}\left(4\frac{t_i}{T_0} - 1\right),$$

so the recombination instant is uniquely defined by the ~~recombination~~ ionization instant.

· We can compute the kinetic energy of the electron at recolision time:

$$E_k(t_r) = \frac{1}{2} m v^2(t_r) = 2U_p\left[\sin(\omega_0 t_r) - \sin(\omega_0 t_i)\right]^2$$



$E_k = 3,17\, U_p$ - maximum kinetic energy of the electron at recompination time.

Cutoff energy $E = I_p + 3,17\, U_p = \hbar\omega_{max}$

we know that $U_p = \frac{E_0}{4\omega^2}$, so increasing E_0 (intensity of the laser) we can increase the cutoff energy.

In practice, it is possible to vary the ratio between long and short trajectories thus changing the parameters of emmitted photons.

Odd harmonics rule:

for the classical model we obtained a set of numbers t_i, t_r which characterize the ionization and recollision time.

This set of numbers $\omega_0 t_i$ and $\omega_0 t_r$ makes $x(t_{i,r})=0$

Then $(\omega_0 t_i + m\pi, \omega_0 t_r + m\pi)$ is also the solution, m is integer.

For given m, we have a trajectory $X_m(t)$,

we can show that $X_m(t) = (-1)^m X_{m=0}(t - m\pi)$

The harmonics are emmited each half cycle π. with alernating phase

Then the emitted field can be expressed as

$$E(t) = + \dots F_h(t + 2\pi/\omega_0) - F_h(t + \pi/\omega_0) + F_h(t) - F_h(t - \pi/\omega_0) + \dots$$

if we find the Fourier transform $\tilde{E}(\omega) = \hat{\mathcal{F}}[E(t)]$,

the only odd multiplyes of ω_0 will survive.

The HHG spectrum contains odd harmonics only.

Semiclassical HHG model (Lewenstein model)

In the lecture on strong field ionization we discussed that the time-dependent WF of the ionized system is:

$$|\Psi(t)\rangle = e^{iI_p t/\hbar}\left(a_0(t)|0\rangle + \int d\vec{k}\, c(\vec{k},t)|\vec{k}\rangle\right)$$

We showed that amplitudes $c(p,t)$ are:

$$a(p,t) = -\frac{i}{\hbar}\int_{t_i}^{t_f} dt'\, E(t')\underbrace{\langle e^{i(\vec{k}+A(t))\cdot\vec{x}}|\vec{x}|\psi_0(t_i)\rangle}_{d[p+A(t)]} e^{-i(S(t)+I_p t)} \qquad S(t) = \int d\tau\,[p+A(t)]^2/2$$

Ignoring the continuum-continuum contributions, the dipole moment can be written as:

$$\langle\Psi(t)|\vec{x}|\Psi(t)\rangle = \int d\vec{k}\; c_0^*(t)\,c(\vec{k},t)\langle 0|\vec{x}|k\rangle + \text{c.c.} =$$

$$= -\frac{i}{\hbar}\int_{t_i}^{t_f} dt'\int d\vec{k}\; c_0^*(t)\, d^*[\vec{k}+A(t)]\, e^{-i(S(t')+I_p t')}\, a_0(t')E(t')\, d[p+A(t')]$$

To get the emmited spectra we need to compute Fourier transform of $\langle\Psi(t)|\hat{j}|\psi(x)\rangle$